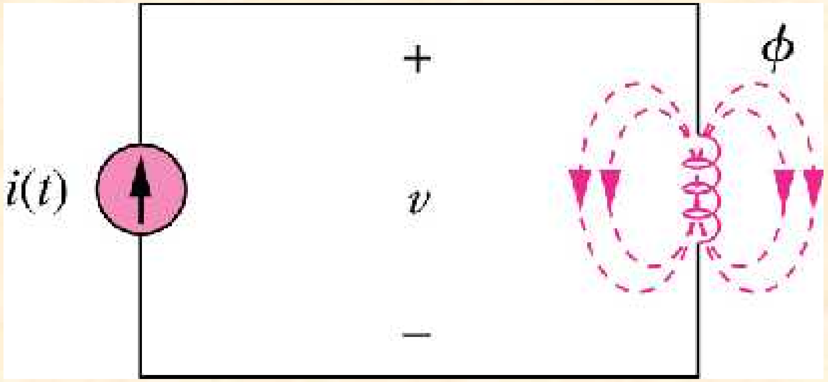
Main Outlines

- Review of self inductance
- Concept of mutual inductance
- Mutual inductance in terms of self inductance
- Polarity of the mutually induced voltages (Dot Convention)
- Procedure for determining dot marking
- Use of dot markings in circuit analysis

Self Inductance (Summary)

$$¤ = \frac{(N\,i)}{\Re} = (N\,i)\,P$$



$$ª = N\,¤ = L\ i$$

Magnetic flux produced by a single coil

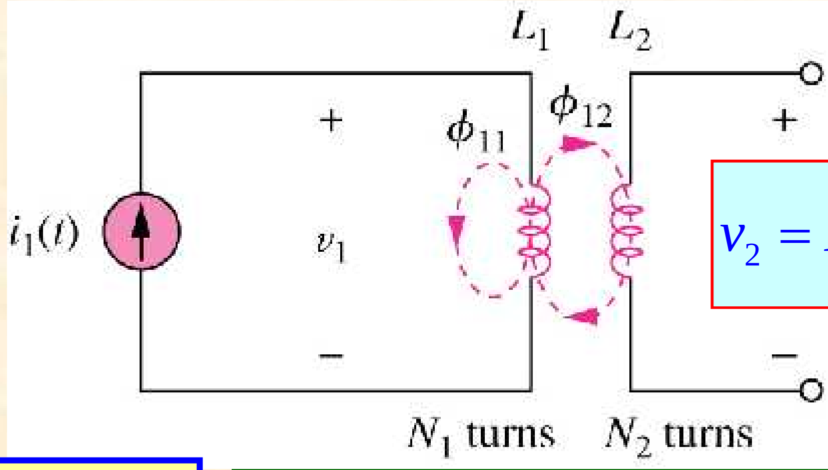
$$v = \frac{dª}{dt} \quad \begin{cases} v = N\dfrac{d¤}{dt} \\[2ex] v = L\dfrac{di}{dt} \end{cases}$$

$$L = \frac{N^2}{\Re} = N^2\,P$$

Mutual Inductance (Summary)

$$v_1 = N_1 \frac{d\phi_1}{dt}$$

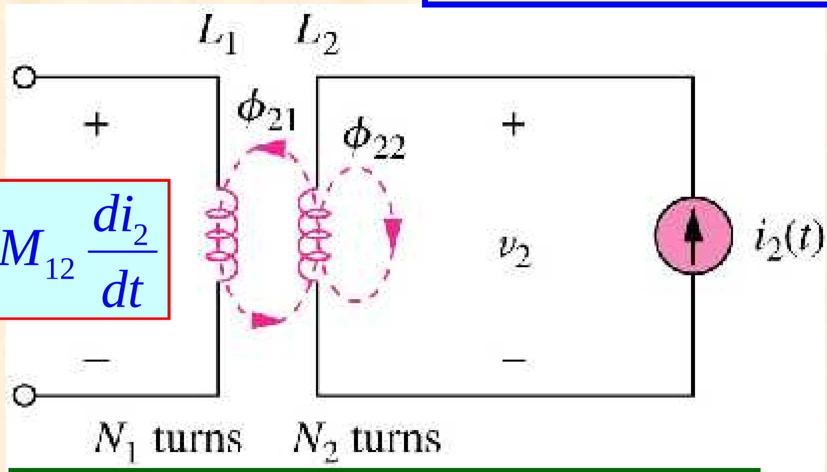
$$v_1 = L_1 \frac{di_1}{dt}$$



$$v_2 = M_{21} \frac{di_1}{dt}$$

$$M_{21} = M_{12} = M$$

Mutual inductance M_{21} of coil 2 with respect to coil 1



$$v_1 = M_{12} \frac{di_2}{dt}$$

$$v_2 = N_2 \frac{d\phi_2}{dt}$$

$$v_2 = L_2 \frac{di_2}{dt}$$

Mutual inductance of M_{12} of coil 1 with respect to coil 2

Mutual inductance in terms of self inductances (Summary)

$$L_1 = N_1^2 \, P_1$$

$$P_1 = \, P_{11} + P_{21}$$

$$L_2 = N_2^2 \, P_2$$

$$P_2 = \, P_{22} + P_{12}$$

$$M = N_1 N_2 \, P_{21}$$

$$L_1 L_2 = M^2 \left(1 + \frac{P_{11}}{P_{12}}\right)\left(1 + \frac{P_{22}}{P_{12}}\right)$$

$$\frac{1}{k^2} = \left(1 + \frac{P_{11}}{P_{12}}\right)\left(1 + \frac{P_{22}}{P_{12}}\right)$$

$$M = k\sqrt{L_1 L_2}$$

“k” is called the coupling coefficient

Coupling Coefficient (Summary)

➢ The coupling coefficient **"k"** is a measure of the percentage of flux from one coil that links another coil

$$\boxed{k = \frac{M}{\sqrt{L_1 L_2}}}$$

❑ **Range of k : $0 \le k \le 1$**

➢ If **k > 0.5** , the coils are said to be **tightly coupled**

➢ If **k < 0.5**, the coils are said to be **loosely coupled**

➢ **k = 0** means the two coils are **not coupled**

➢ **k = 1** means the two coils are **perfectly coupled**

k can be expressed in terms of flux as

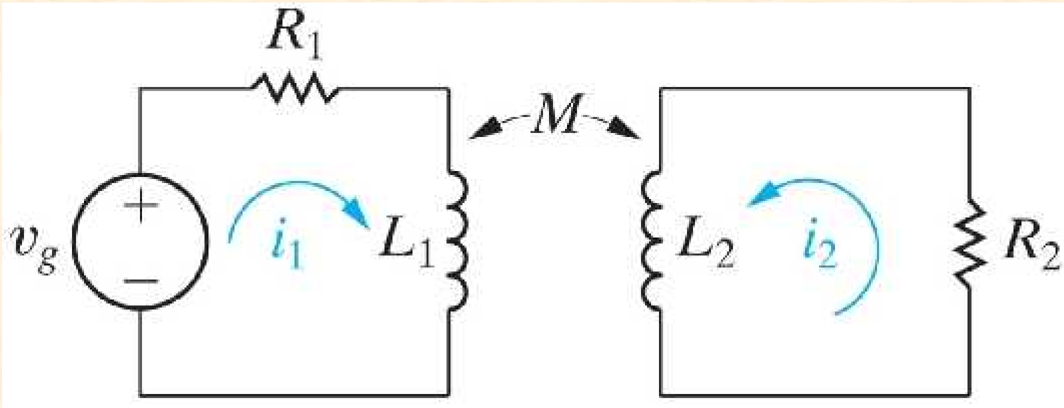
$$k = \frac{\Phi_{12}}{\Phi_{11} + \Phi_{12}}$$

or $$k = \frac{\Phi_{21}}{\Phi_{21} + \Phi_{22}}$$

$k = 1$ means perfect coupling.

$$\Rightarrow \Phi_{11} = \Phi_{22} = 0$$

Magnetically Coupled Circuits



- ❑ There will be **two voltages** across each coil;
- ✓ **"self-induced" voltage**, **L(di/dt)**, and
- ✓ **"mutually induced" voltage**, **M(di/dt)**

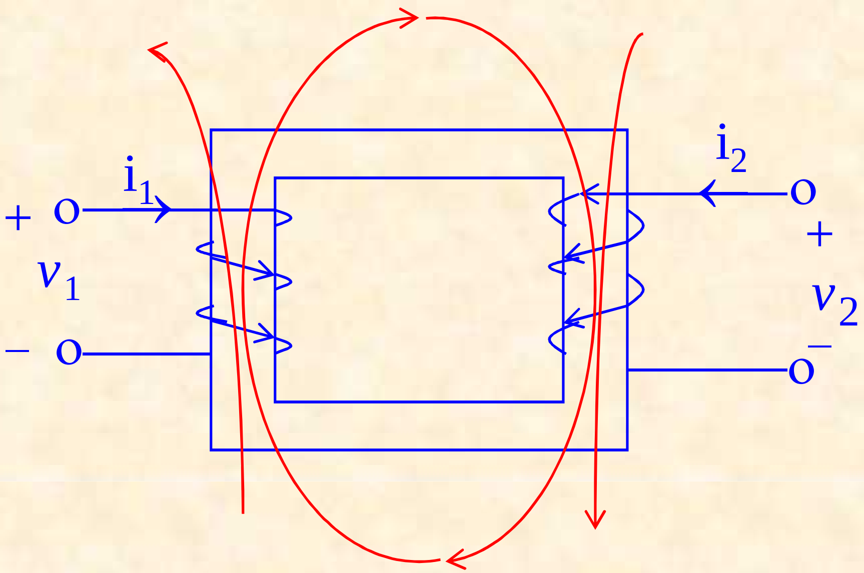
➢ The polarity of the self-induced voltage is the same as the **resistive voltage drop**

➢ The polarity of the mutually induced voltage can be determined according to **Lenz's Law**

Magnetically Coupled Circuits

$$a_1(t) = L_1 i_1(t) + M_{12} i_2(t)$$

$$a_2(t) = M_{21} i_1(t) + L_2 i_2(t)$$



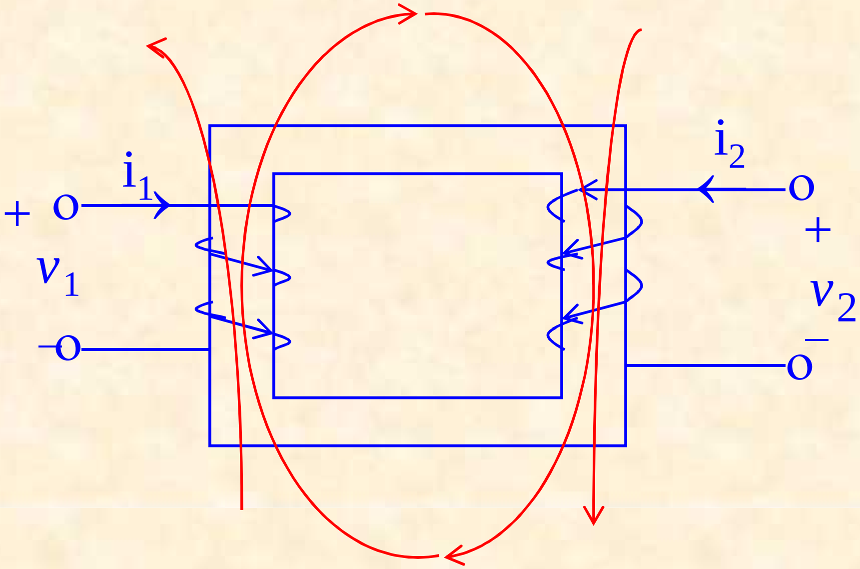
$$v_1 = \frac{d a_1}{dt} = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

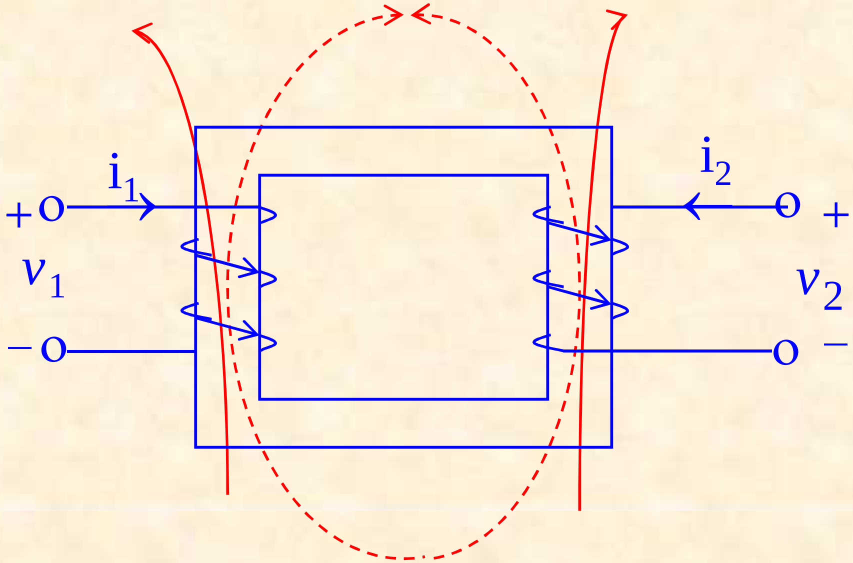
$$v_2 = \frac{d a_2}{dt} = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

Mutual induced voltage

Self induced voltage

Magnetically Coupled Circuits





$$\lambda_1(t) = L_1 i_1(t) + M i_2(t)$$

$$\lambda_2(t) = M i_1(t) + L_2 i_2(t)$$

$$v_1 = \frac{d\lambda_1}{dt} = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2 = \frac{d\lambda_2}{dt} = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

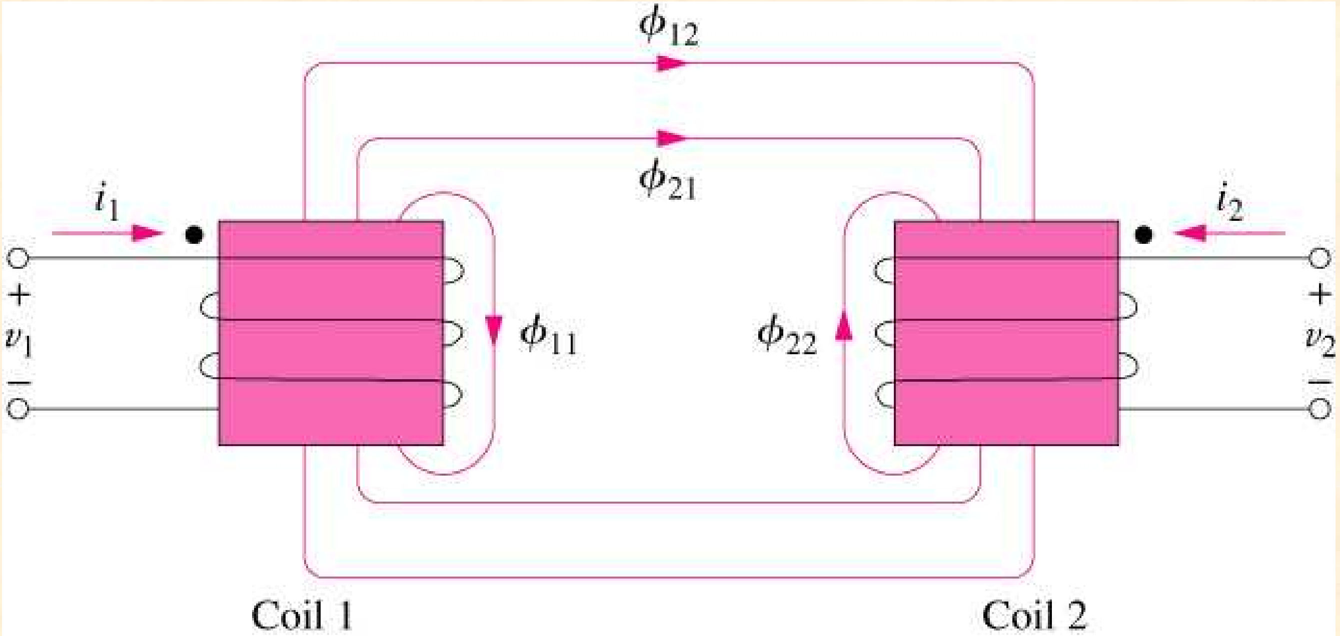
$$\lambda_1(t) = L_1 i_1(\mathrm{t}) - M i_2(t)$$

$$\lambda_2(t) = -M i_1(t) + L_2 i_2(t)$$

$$v_1 = \frac{d\lambda_1}{dt} = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

$$v_2 = \frac{d\lambda_2}{dt} = -M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

Magnetically Coupled Circuits



i_1 induces ϕ_{11} and ϕ_{12},
i_2 induces ϕ_{21} and ϕ_{22}.

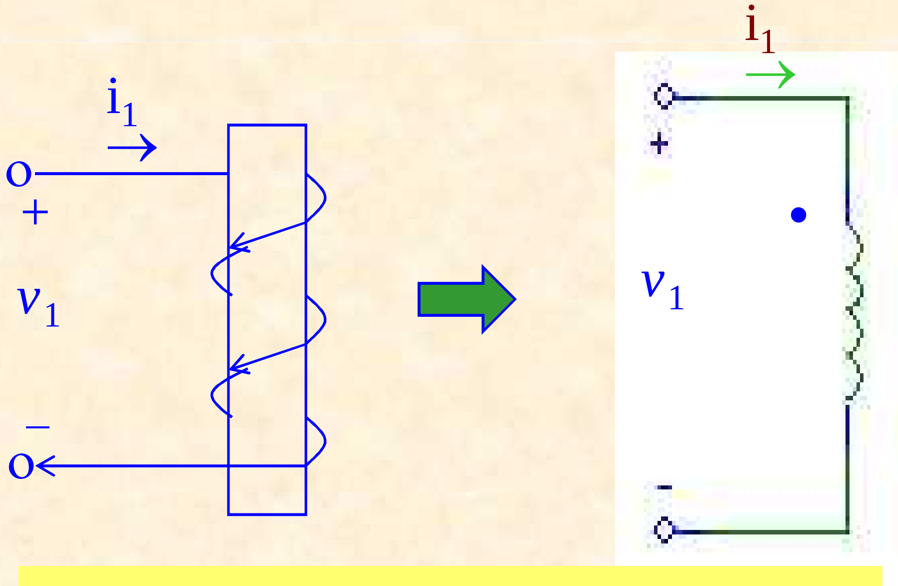
$$\phi_1 = (\phi_{11} + \phi_{12}) + \phi_{21}$$
$$\phi_2 = \phi_{12} + (\phi_{21} + \phi_{22})$$

$$v_1 = N_1 \frac{d\phi_1}{dt} = N_1 \frac{d(\phi_{11} + \phi_{12})}{dt} + N_1 \frac{d\phi_{21}}{dt} = L_1 \frac{di_1}{dt} + M_{12} \frac{di_2}{dt}$$

$$v_2 = N_2 \frac{d\phi_2}{dt} = N_2 \frac{d(\phi_{21} + \phi_{22})}{dt} + N_2 \frac{d\phi_{12}}{dt} = L_2 \frac{di_2}{dt} + M_{21} \frac{di_1}{dt}$$

Dot Convention

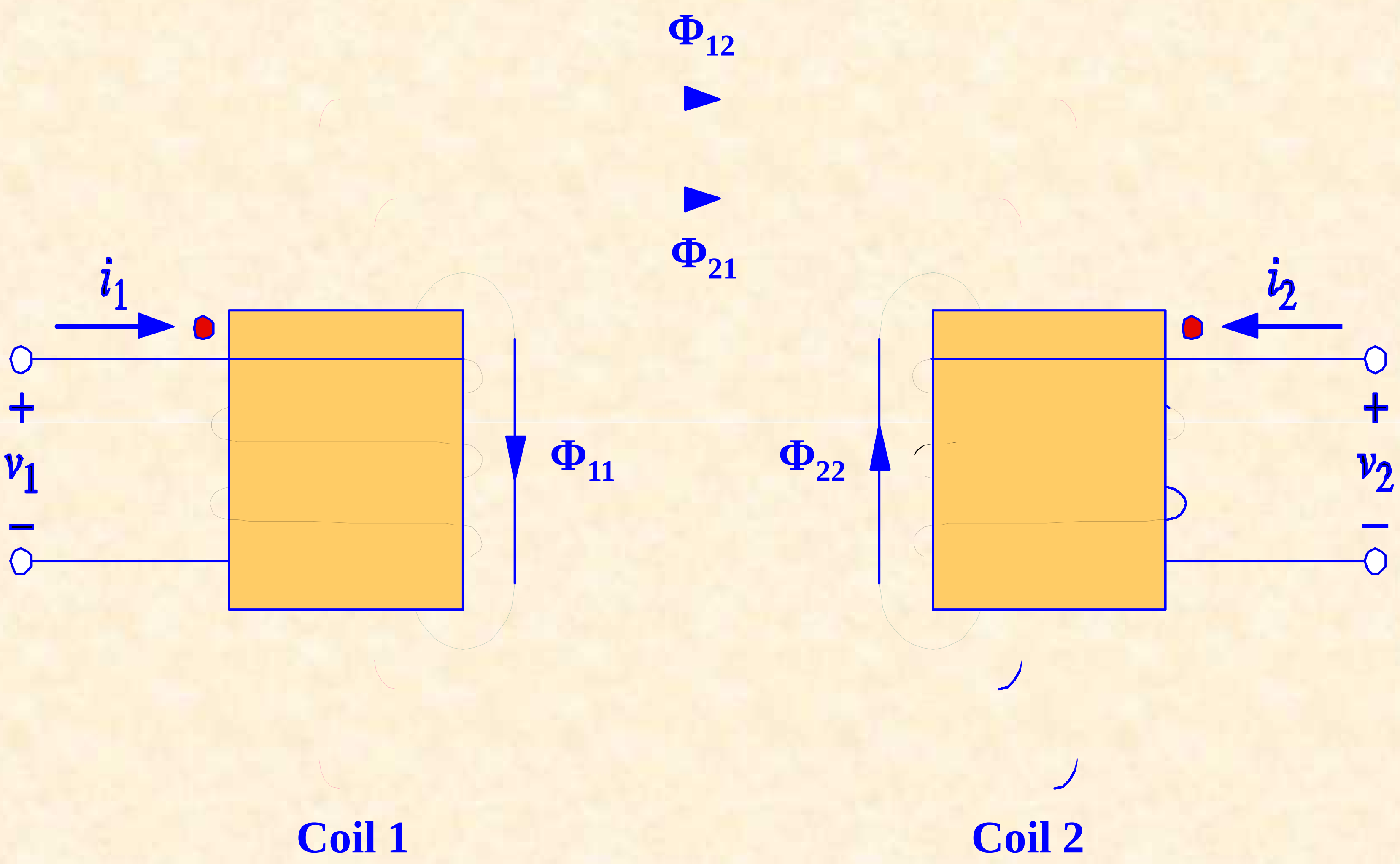
- Required to determine polarity of "mutual" induced voltage
- A **dot** is placed in the circuit at one end of each of the two magnetically coupled coils <u>to indicate the direction of the magnetic flux</u> if current enters that dotted terminal of the coil



Dot indicate the direction in which the coils are wound

Lumped Coil Representation

Dot Convention



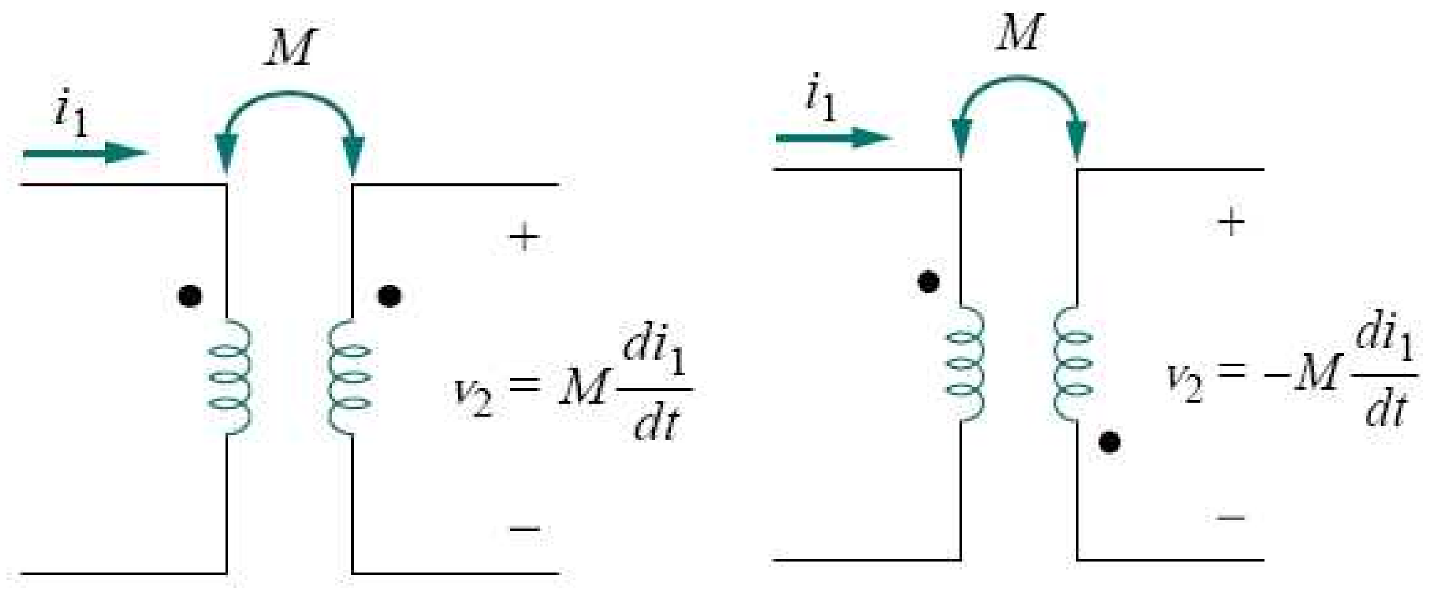
Dot Convention

- ❑ Dot convention is stated as follows:
- ✓ if a current ENTERS the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is POSITIVE at the dotted terminal of the second coil

- ❑ Conversely, Dot convention may also be stated as follow:
- ✓ if a current LEAVES the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is NEGATIVE at the dotted terminal of the second coil

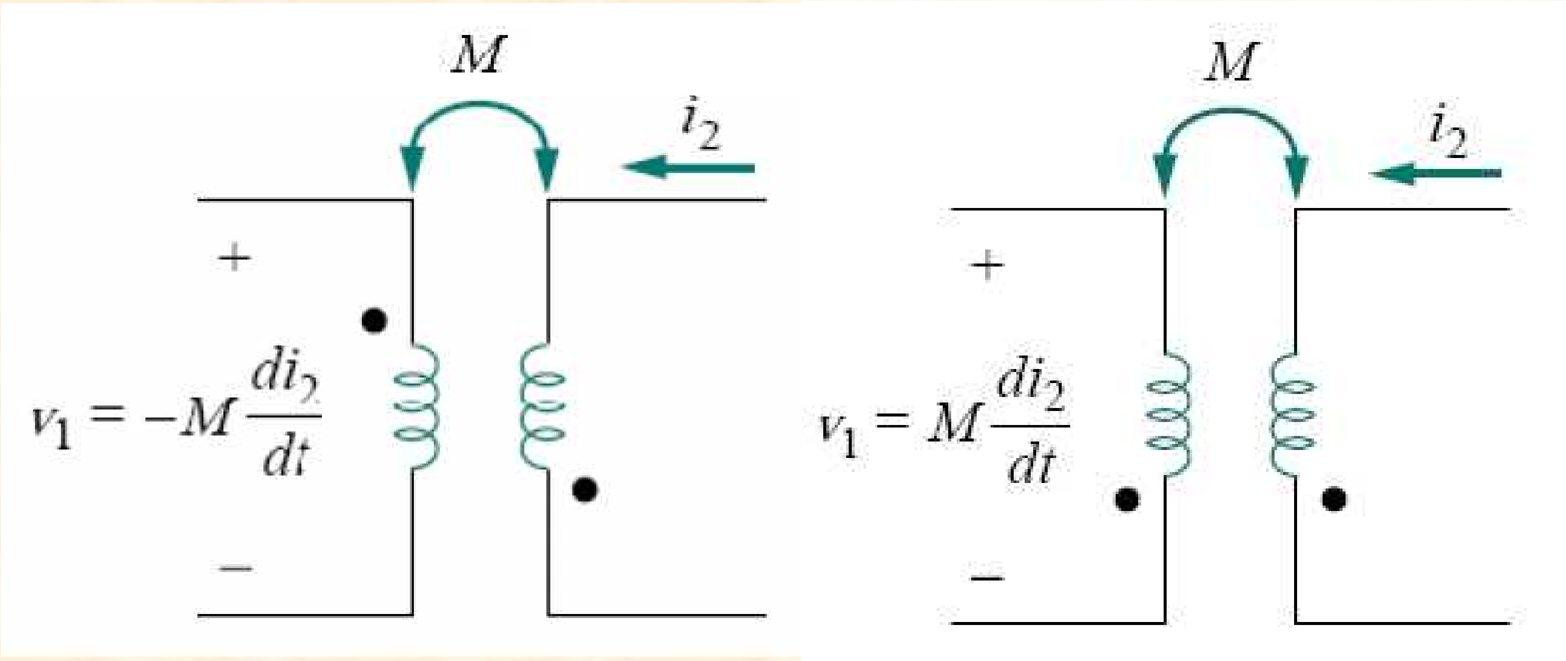
Dot Convention

If a current enters the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is positive at the dotted terminal of the second coil.

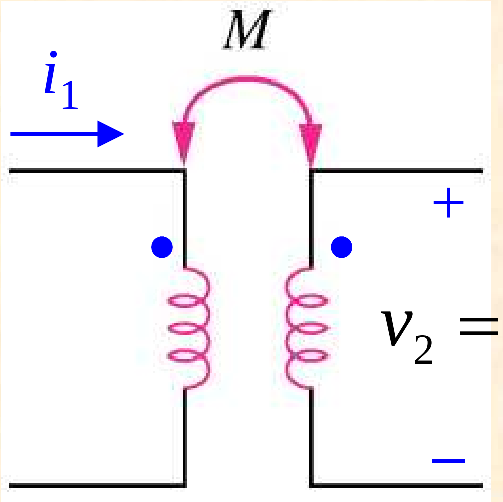


Dot Convention

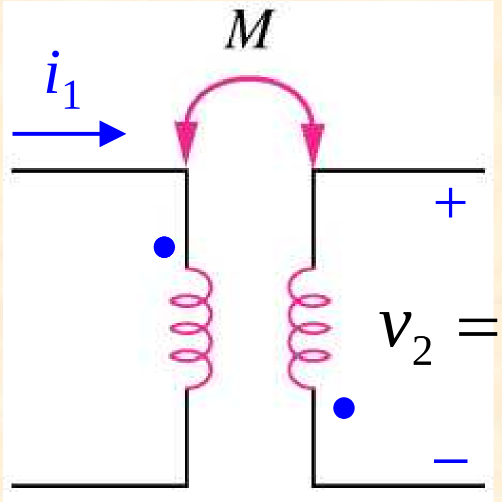
If a current leaves the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is negative at the dotted terminal of the second coil.



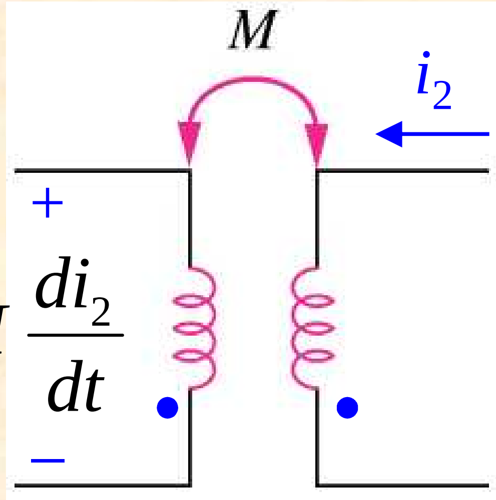
Dot Convention



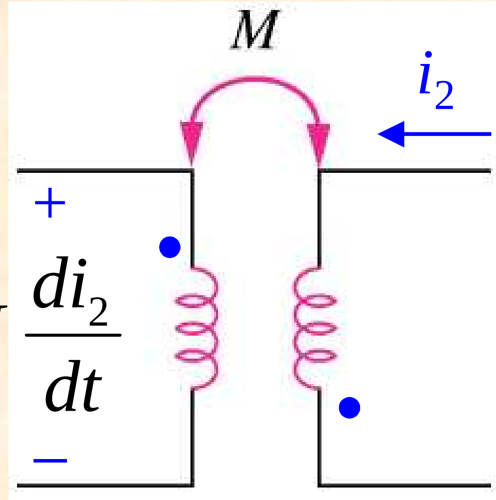
$$v_2 = M\frac{di_1}{dt}$$



$$v_2 = -M\frac{di_1}{dt}$$



$$v_1 = M\frac{di_2}{dt}$$



$$v_1 = -M\frac{di_2}{dt}$$

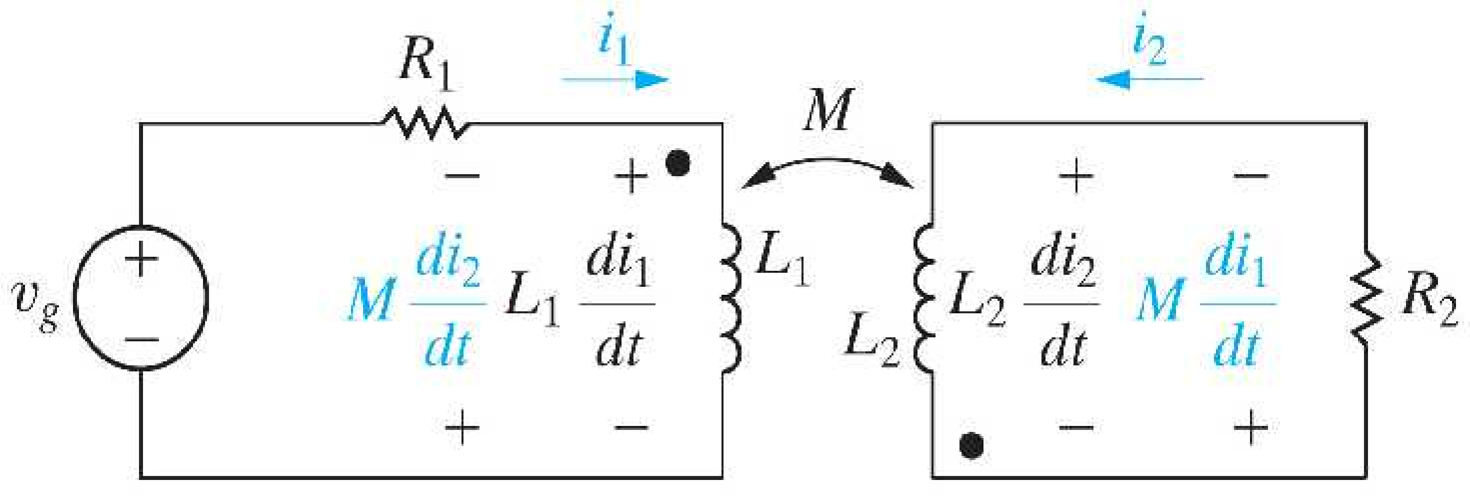
The Rule for using the Dot Convention

- The following dot rule may be used:

- ✓ When the assumed currents both entered or both leaves a pair of coupled coils by the dotted terminals, the signs on the **L – terms** will be **the same** as the signs on the **M – terms**

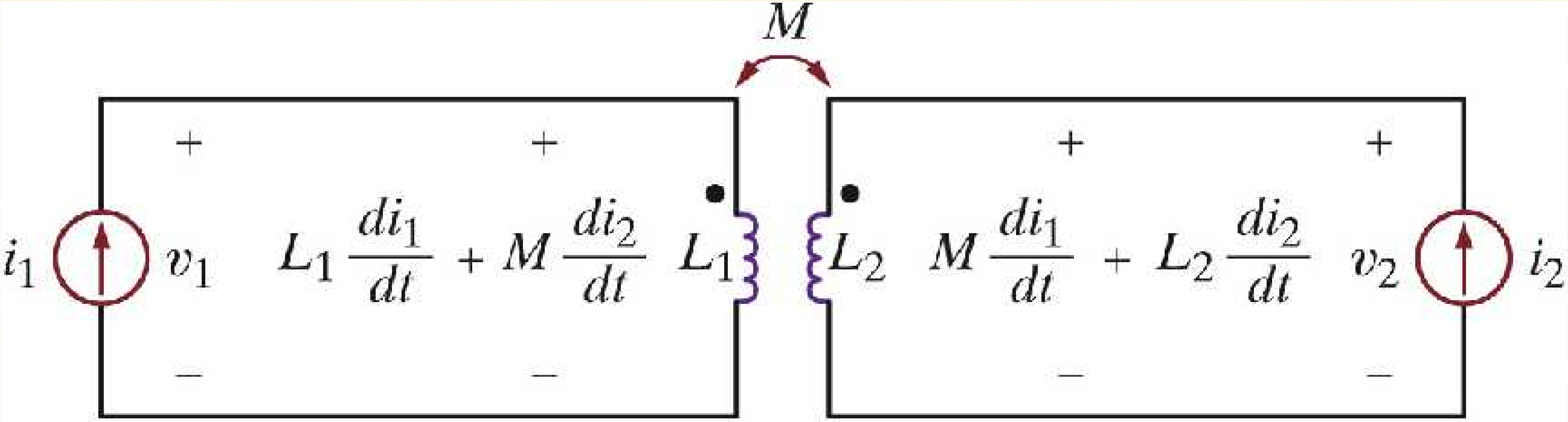
- ✓ If one current enters by a dotted terminals while the other leaves by a dotted terminal, the sign on the **M – terms** will be **opposite** to the signs on the **L – terms**.

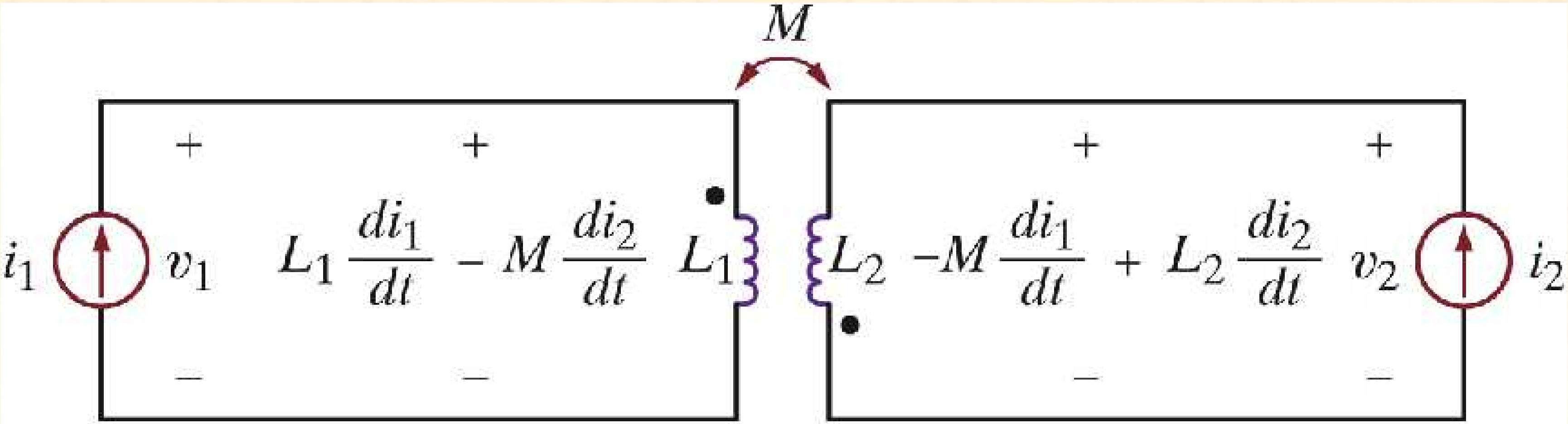
The Rule for Using the Dot Convention



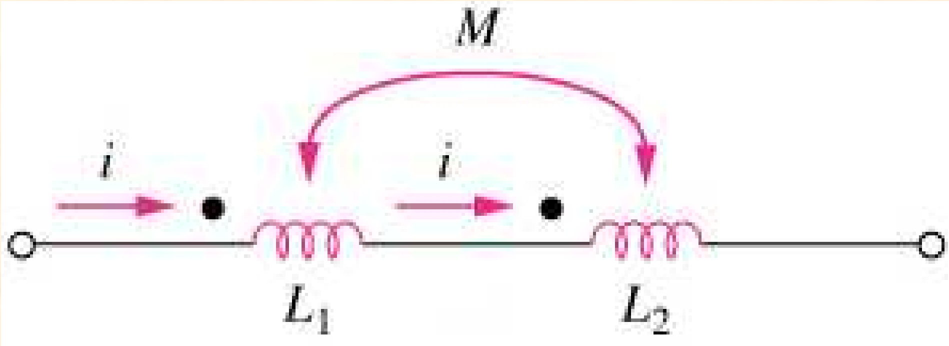
- ✓ The voltage induced in coil 1 by the current in coil 2 is **negative** at the dotted terminal of coil 1
- ✓ The voltage induced in coil 2 by the current in coil 1 is **positive** at the dotted terminal of coil 2

The Rule for Using the Dot Convention





Dot Convention for Coils in Series



$+ \quad v_1 \quad - \qquad + \quad v_2 \quad -$

$$L = L_1 + L_2 + 2M$$

(series-aiding connection)

$$v_1 = L_1 \frac{di}{dt} + M_{12} \frac{di}{dt}$$

$$v_2 = L_2 \frac{di}{dt} + M_{21} \frac{di}{dt}$$

$$v = v_1 + v_2$$

$$= L_1 \frac{di}{dt} + M_{12} \frac{di}{dt} + L_2 \frac{di}{dt} + M_{21} \frac{di}{dt}$$

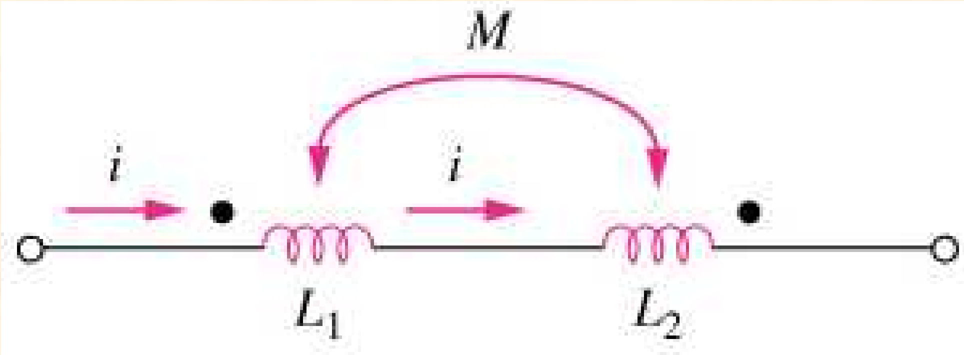
$$= \left(L_1 + L_2 + M_{12} + M_{21}\right) \frac{di}{dt}$$

But $M_{12} = M_{21} = M$,

$$\Rightarrow v = \left(L_1 + L_2 + 2M\right) \frac{di}{dt}$$

$$\Rightarrow \boxed{L_{eq} = L_1 + L_2 + 2M}$$

Dot Convention for Coils in Series



$+ \quad v_1 \quad - \qquad + \quad v_2 \quad -$

$$L = L_1 + L_2 - 2M$$
(series - opposition connection)

$$v_1 = L_1 \frac{di}{dt} - M_{12}\frac{di}{dt}$$

$$v_2 = L_2 \frac{di}{dt} - M_{21}\frac{di}{dt}$$

$$v = v_1 + v_2$$

$$= L_1 \frac{di}{dt} - M_{12}\frac{di}{dt} + L_2 \frac{di}{dt} - M_{21}\frac{di}{dt}$$

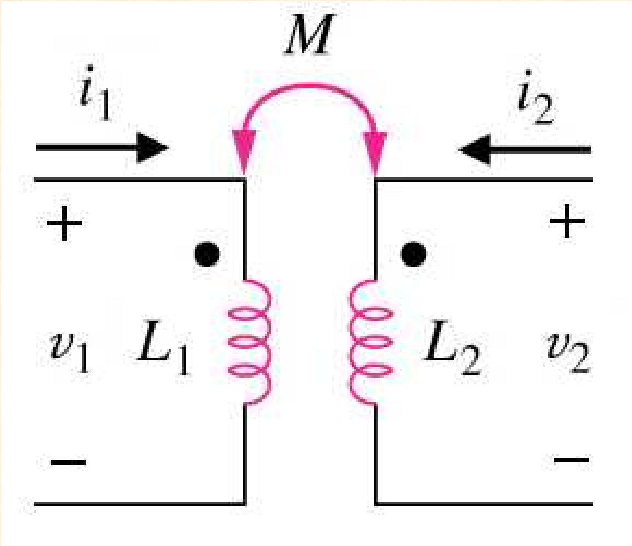
$$= \left(L_1 + L_2 - M_{12} - M_{21}\right)\frac{di}{dt}$$

But $M_{12} = M_{21} = M$,

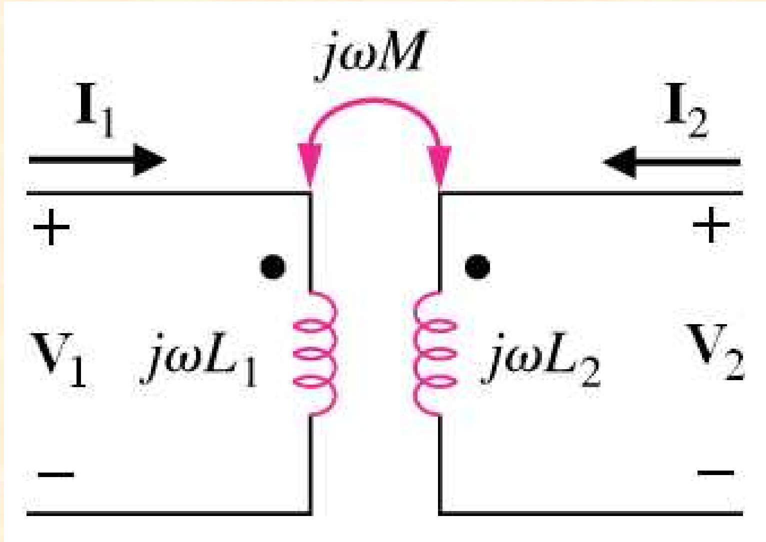
$$\Rightarrow v = \left(L_1 + L_2 - 2M\right)\frac{di}{dt}$$

$$\Rightarrow \boxed{L_{eq} = L_1 + L_2 - 2M}$$

Circuit Model for Coupled Inductors



Time-domain circuit



Frequency-domain circuit

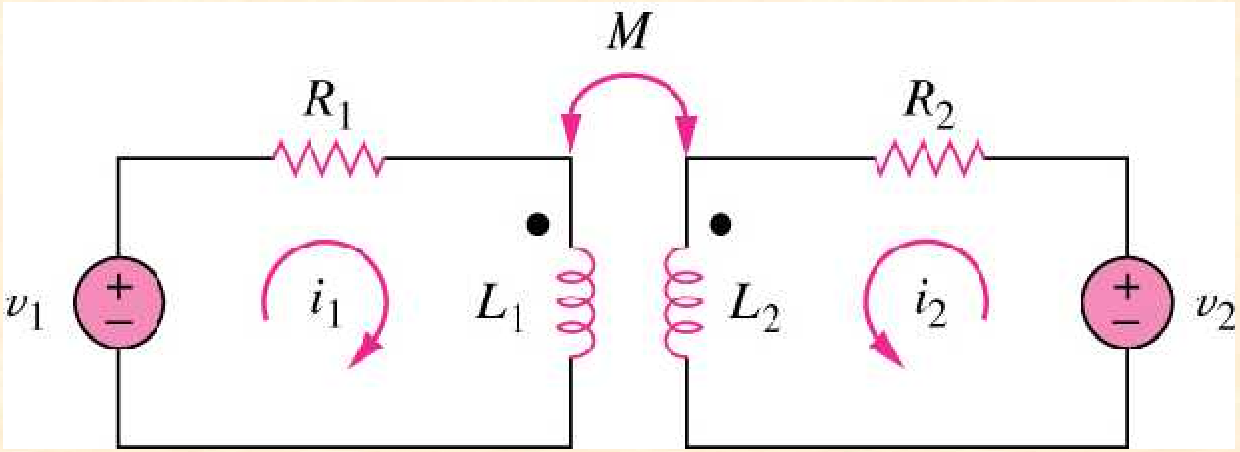
$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

$$\mathbf{V}_1 = j\mu L_1 \mathbf{I}_1 + j\mu M \mathbf{I}_2$$

$$\mathbf{V}_2 = j\mu M \mathbf{I}_1 + j\mu L_2 \mathbf{I}_2$$

Mesh Equations using Dot Convention



Applying KVL to mesh 1 gives

$$v_1 = i_1 R_1 + L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

Applying KVL to mesh 2 gives

$$v_2 = i_2 R_2 + L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

In phasor (frequency) domain

$$\mathbf{V}_1 = (R_1 + j\mu L_1)\mathbf{I}_1 + j\mu M \mathbf{I}_2$$

$$\mathbf{V}_2 = j\mu M \mathbf{I}_1 + (R_2 + j\mu L_2)\mathbf{I}_2$$